PERTH MODERN SCHOOL
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Independent Public School

Course: Methods Year: 11

Student Name: **Marking Key** Teacher Name: ____________

Date: 09/09/22

Task Type: **Response**

Time Allowed: **40 minutes**

Number of Questions: **8**

Materials Required: CAS calculator (ClassPad) and one double-sided A4 pages of notes (to be provided by the student)

Standard Items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler and highlighters

Special Items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper (both sides) and up to three calculators approved for use in the WACE examinations

Marks Available: **40 marks**

Task Weighting: **10 %**

Formula Sheet Provided: Yes

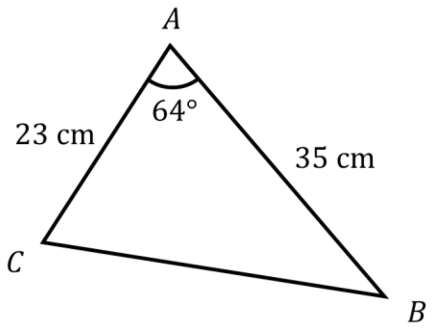
Do not penalise for missing/incorrect units.
Do not penalise for rounding to the incorrect number of decimal places.

Note: All questions worth more than 2 marks require working to obtain full marks.

TEST 4: TRIGONOMETRY AND EXPONENTIALS

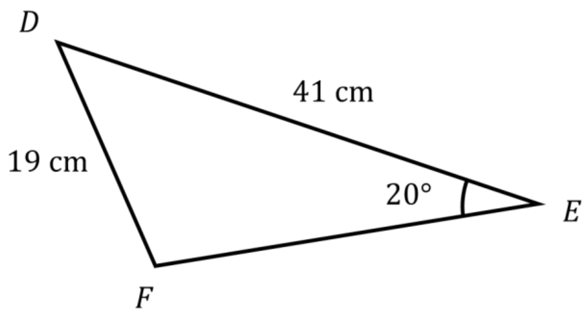
Question 1 [6 marks – 2, 2, 2] (1.2.4)

a) Determine BC, to 1 decimal place.



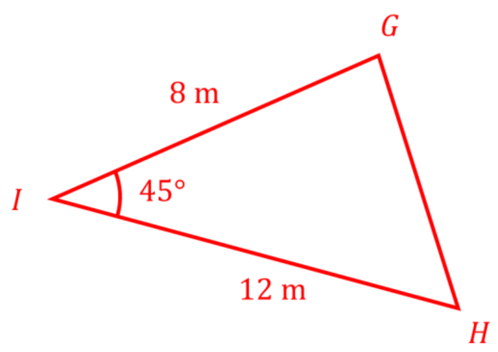
Solution	
$BC^2 = 23^2 + 35^2 - 2(23)(35)\cos 64°$ $BC = 32.4 \text{ cm}$	
Specific behaviours	
✓	Substitutes into cosine rule
✓	Calculates length

b) Determine $\angle DFE$, to the nearest degree.



Solution	
$\frac{\sin \angle DFE}{41} = \frac{\sin 20°}{19}$ $\angle DFE = 48°, 132°$	
Specific behaviours	
✓	Substitutes into sine rule
✓	Calculates both possible angles

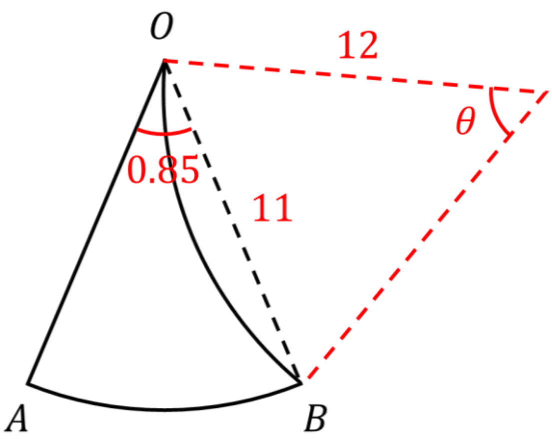
c) Find the exact area of ΔGHI, given that $GI = 8$ m, $HI = 12$ m and $\angle GIH = 45°$.



Solution	
$\text{Area} = \frac{1}{2}(8)(12)\sin 45°$ $= 24\sqrt{2} \text{ m}^2$	
Specific behaviours	
✓	Substitutes into area formula
✓	Calculates exact area

Question 2 **[4 marks]** (1.2.5-1.2.6)

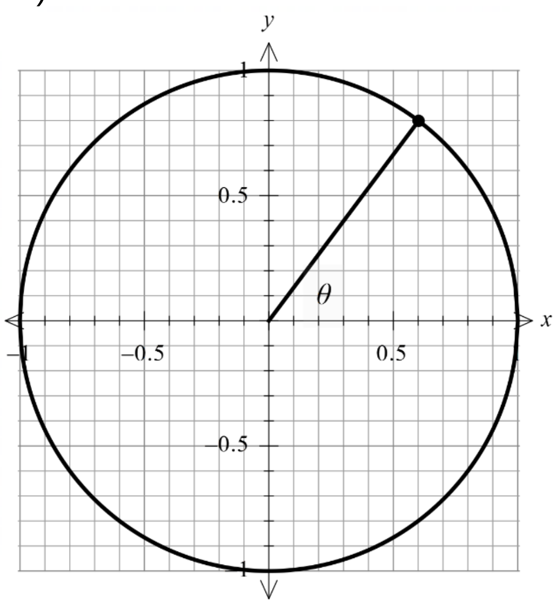
For the shape below, arc AB has radius 11 cm, arc OB has radius 12 cm and $\angle AOB = 0.85$. Find the area of the shape to 1 decimal place.



Solution
$11 = 2(12)\sin\frac{\theta}{2}$ $\theta = 0.95$ $A = \frac{1}{2}(11)^2(0.85) - \frac{1}{2}(12)^2(0.95 - \sin 0.95)$ $= 41.5 \text{ cm}^2$
Specific behaviours
✓ Substitutes into chord formula   ✓ Calculates angle subtended by arc OB   ✓ Substitutes into sector and segment area formulas   ✓ Calculates area (difference)   *Award 1 mark for finding sector area AOB*

Question 3 **[4 marks – 2, 2]** (1.2.7-1.2.8)

a) Consider the unit circle below.



i) Find $\cos(180° + \theta)$ to 1 decimal place.

Solution
$\cos(180° + \theta) = -\cos\theta$ $= -0.6$
Specific behaviours
✓ States value

ii) Find $\sin(-\theta)$ to 1 decimal place.

Solution
$\sin(-\theta) = -\sin\theta$ $= -0.8$
Specific behaviours
✓ States value

b) Determine the exact values of the following:

i) $\sin 135°$

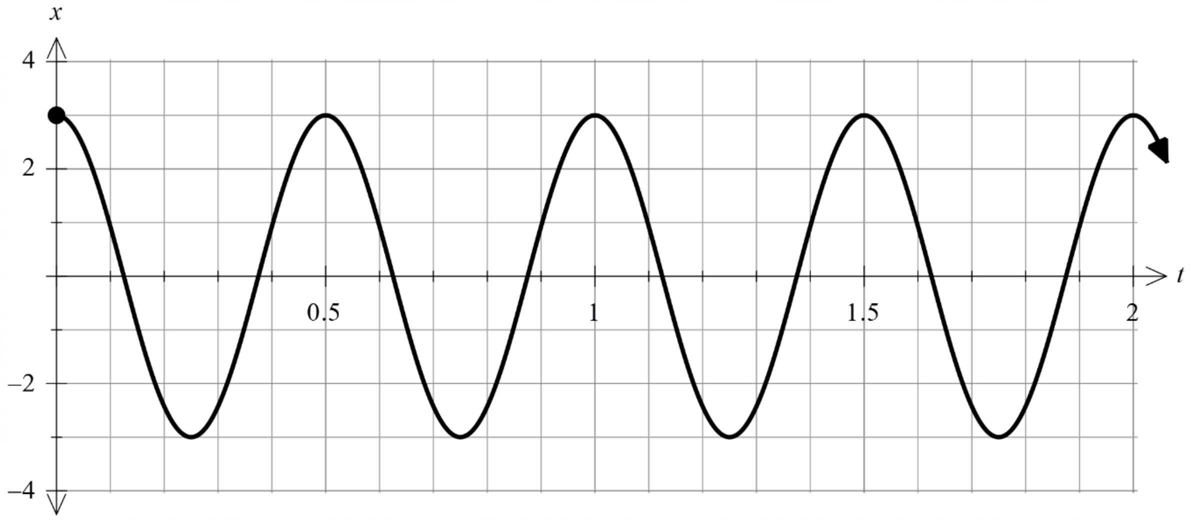
Solution
$\sin 135° = \frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$
Specific behaviours
✓ States value

ii) $\tan 300°$

Solution
$\tan 300° = -\sqrt{3}$
Specific behaviours
✓ States value

Question 4 **[3 marks – 1, 2]** (1.2.9-1.2.12, 1.2.15)

A pendulum oscillates such that its horizontal position x cm with respect to time t seconds is as shown in the graph below.



a) State the amplitude and period of the pendulum.

Solution
Amplitude $= 3$ cm Period $= 0.5$ seconds
Specific behaviours
✓ States amplitude and period

b) Given that $x(t) = a\cos(bt)$, state the equation of the pendulum's motion.

Solution
$x(t) = 3\cos(4\pi t)$ or $3\cos(720t)$
Specific behaviours
✓ Determines value of a ✓ Determines value of b *Accept if equation is not stated*

Question 5 **[7 marks – 3, 4]** (1.2.16, 1.2.14)

a) Given that $\sin a = b$, where a is a positive acute angle, determine the exact solutions of $\sin 2\theta = -b$ where $0 \leq \theta \leq 2\pi$.

Solution
$2\theta = \pi + a,\ 2\pi - a,\ 3\pi + a,\ 4\pi - a$ $\theta = \frac{\pi + a}{2}, \frac{2\pi - a}{2}, \frac{3\pi + a}{2}, \frac{4\pi - a}{2}$
Specific behaviours
✓ States first two solutions of 2θ ✓ States second two solutions of 2θ ✓ Divides by 2 to determine solutions of θ

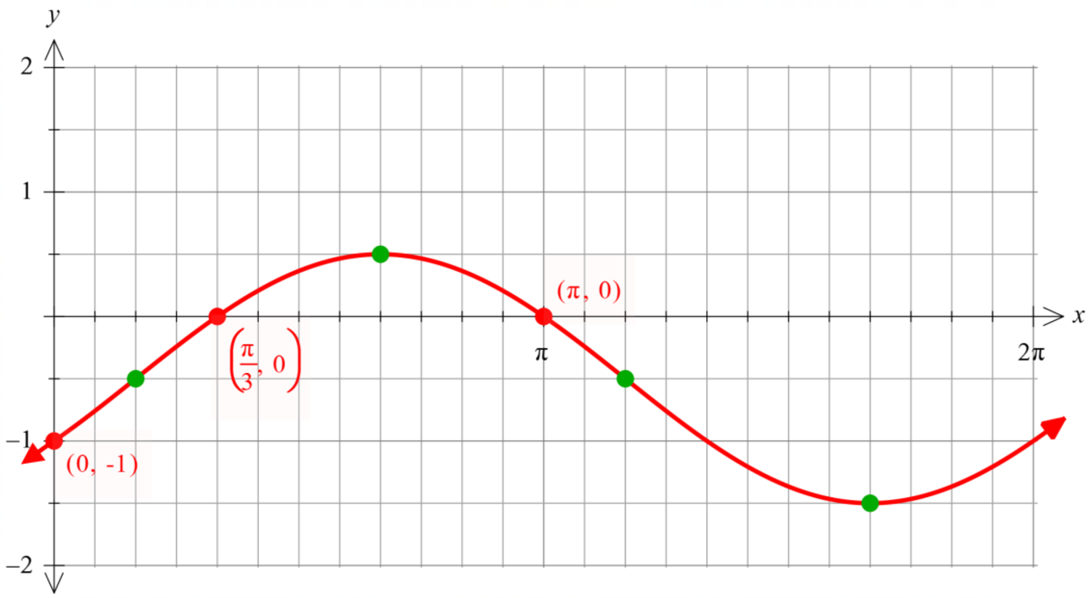
Question 5 (continued)

b) If $\cos A = -\frac{12}{13}$ where $180° < A < 270°$ and $\sin B = \frac{15}{17}$ where B is obtuse, determine the exact value of $\cos(A - B)$.

Solution
$\sin A = -\frac{5}{13}$ $\cos B = -\frac{8}{17}$ $\cos(A - B) = \cos A \cos B + \sin A \sin B$ $= \left(-\frac{12}{13}\right)\left(-\frac{8}{17}\right) + \left(-\frac{5}{13}\right)\left(\frac{15}{17}\right)$ $= \frac{21}{221}$
Specific behaviours
✓ Determines exact value of $\sin A$ or the correct magnitudes of $\sin A$ and $\cos B$ ✓ Determines exact value of $\cos B$ or the correct signs of $\sin A$ and $\cos B$ ✓ Substitutes into identity ✓ Calculates exact value of $\cos(A - B)$

Question 6 **[4 marks]** (1.2.9-1.2.12)

Graph $y = \sin\left(x - \frac{\pi}{6}\right) - \frac{1}{2}$ on the axes below, labelling the exact coordinates of all intercepts.



Specific behaviours
✓ Passes through points marked in green ✓ Correct y-intercept *(accept if not labelled)* ✓ Correct x-intercepts *(accept if not labelled)* ✓ Correct general shape *Follow through up to 2 out of 4 marks from any incorrect transformations.*

Question 7 [6 marks – 3, 3] (2.1.1-2.1.2, 2.1.7)

a) Simplify $(64a^6b^{15})^{\frac{1}{3}} \div (a^5bc^2)$, expressing your answer with positive indices.

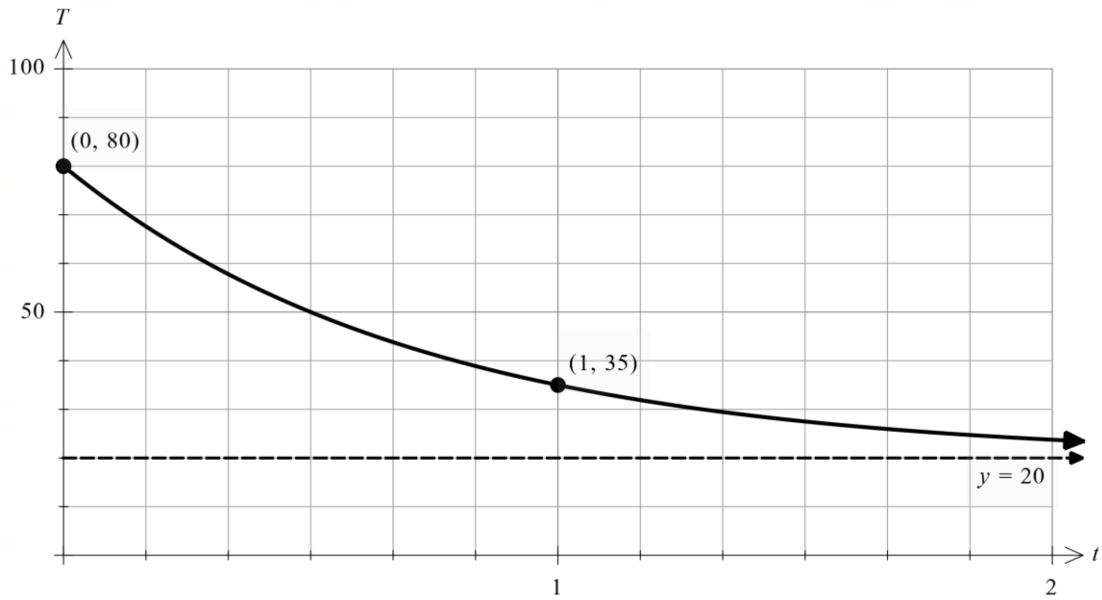
Solution
$(64a^6b^{15})^{\frac{1}{3}} \div (a^5bc^2) = 4a^2b^5 \div (a^5bc^2)$ $= 4a^{-3}b^4c^{-2}$ $= \frac{4b^4}{a^3c^2}$
Specific behaviours
✓ Expands brackets   ✓ Divides to combine variables   ✓ Expresses with positive indices

b) Solve $16^x = 128$ for the exact value of x, showing all working.

Solution
$(2^4)^x = 2^7$ $2^{4x} = 2^7$ $4x = 7$ $x = \frac{7}{4}$
Specific behaviours
✓ Expresses using a base of 2   ✓ Equates indices   ✓ Solves for exact value of x

Question 8 [6 marks – 4, 2] (2.1.1-2.1.2, 2.1.7)

A cup of green tea is poured at 80°C and cools down towards room temperature at an exponential rate, as shown below.



a) The temperature T°C after t hours can be modelled using the equation $T = ab^t + k$. Using the information shown, determine the equation.

Solution
From asymptote $y = k$: $k = 20$ $T = ab^t + 20$ From y-intercept $(0, 80)$: $80 = ab^0 + 20$ $a = 60$ $T = 60b^t + 20$ From $(1, 35)$: $35 = 60b^1 + 20$ $b = 0.25$ $T = 60(0.25)^t + 20$
Specific behaviours
✓ Determines value of k ✓ Determines value of a ✓ Substitutes $(1, 35)$ ✓ Determines value of b *Accept if equation is not stated*

b) The safe drinking temperature is estimated to be about 57°C. How long does the tea need to cool for to be safe to drink, to the nearest minute?

Solution
Substituting in $T = 57$: $57 = 60(0.25)^t + 20$ $t = 0.349$ hours $t = 21$ minutes
Specific behaviours
✓ Substitutes into equation ✓ Determines time to the nearest minute *If estimated from graph, award 1 mark for 20 minutes or 2 marks for 21 minutes*

End of Test

SUPPLEMENTARY PAGE

Question: _____

Question: _____